MATH 160 – Probability (Mostly Chapter 5)

Experiments, Outcomes & Sample Space [Sec 5-1]
Events: Simple & Compound [Sec 5-2, but not explained well]]
    Venn Diagram [Sec 5-2,]
    Tree Diagram [Not really in the book, but an example in Sec 5-4,]
Probability
    2 Properties [not fully discussed in book but a bit in Sec 5-1,]
Approaches to Probability [Kind of discussed in the book in Sec 5-1]
    Classical
    Empirical (or relative frequency)
    Subjective
Counting Rule [Not called this, but Sec 5-4]
Conditional Probability [Sec 5-3]
Mutually Exclusive Events [Sec 5-2]
Independent vs. Dependent Events [Sec 5-2]
Complementary Events [Sec 5-2]
Intersection of Events [Book doesn't use this term but discussed in Sec 5-3]
    Probability of an intersection is also called "joint probability"
    Multiplication Rule (& special case for independent events
Union of Events [Book doesn't us this term but discussed in Sec, 5-2]
    Addition Rule (& special case for mutually exclusive events)

Note: Section 5-4 material will be covered with the next unit.

We will be dealing with events and probability of events. These are 2 different things and you need to keep them straight.

There are equations involved in this section for probabilities. You do not, however, have to use them in all situations (sometimes you can logically determine the probabilities). I do expect you to be able to use the equations when required.

General Addition Rule: P(A or B) = P(A) + P(B) – P(A and B)
General Addition Rule for Mutually Exclusive Events: P(A or B) = P(A) + P(B)
General Multiplication Rule: P(A and B) = P(A)P(B|A) = P(B)P(A|B)
General Multiplication Rule for Independent Events: P(A and B) = P(A)P(B)

Compound vs. Simple Events
Simple events are events that consist of only 1 outcome.
Compound events are events that consist of 2 or more outcomes.

Mutually Exclusive Events
Mutually exclusive events are events that have no outcomes in common. This can be true for any number of events.

Statistically Independent Events
Two events are statistically independent if given that one event has occurred, it doesn't affect the probability of the second event occurring. This can be shown a calculation. Two events are statistically independent if any of the following are true (note that is one is true, they are all true):

P(A) = PA|B)
P(B) = P(B|A)
P(A and B) = P(A) x P(B)

Bonus Points

Suppose that a statistics class has 25 students. Let A be the event that 2 or more of these students share the same birthday (day and month, but not year). Ignore Leap Year effects and assume that all 365 possible birthdays are equally likely. For 1 BP, determine P(A). For another BP, give the general form of the equation for any number of students. Hint: It may be helpful to consider complementary events.

1 BP: The principal of a large high school would like to determine the proportion of students in her school who used drugs during the past week. Because the results of directly asking each student “have you used drugs during the past week?” would be unreliable, the principal might use the following randomized response scheme. Each student rolls a fair die once, and only he or she knows the outcome. If a 1 or 2 is rolled, the student must answer the sensitive question truthfully. However, if any other number is rolled, the student must answer the question with the opposite of the true answer (i.e., they must lie). In this way, the principal would not know whether a “yes” response means the student used drugs and answered truthfully or the student did not use drugs and answered untruthfully. If 60% of the students in the school respond “yes”, what proportion of the students actually did use drugs during the past week? (Note: Splitting the outcomes of the roll of the die as described represents only one possible random strategy. The outcomes could be split differently but not in such a way that the probability of answering truthfully is .5. In that case, it would not be possible to estimate the proportion of all students who actually used drugs over the past week.) For another BP, show me why we can’t use .5 for the random split.

For 1 point, tell me why (statistically, that is) the following joke is funny.
Let me take you back to a time before all the security checks of luggage on airplanes. I had a friend who never flew anywhere. When I asked him why, he replied, “because he heard that there was a 1 in 1000 chance that a bomb was on the plane and that those odds made make me feel unsafe”. A little while ago, I saw him at the airport. When I asked him what changed his mind about flying, he replied “I figured out that the chance of 2 bombs being on a plane is 1/1000 times 1/1000, or 1 in a million and I feel safe with those odds” Then he leaned over to me and whispered “so I’m carrying a bomb in my luggage.”

1 BP Famous Game Show Problem: You are on a game show. The host gives you the choice of picking 1 of 3 doors. Behind 1 door is a brand new car. Each of the other 2 doors has a goat behind them. You pick a door but don’t know what is behind it. The host opens one of the other 2 doors to reveal a goat. He then asks you if you want to trade whatever is behind your door for what is behind the last closed door. Tell me your probability of winning the car if you elect to trade doors. Explain your logic.

Homework Chapter 5

1. In a group of adults, some are computer literate, and the others are computer illiterate. If 2 adults are randomly selected from this group, how many total outcomes are possible? Draw a tree diagram for this experiment.

List all the outcomes included in each of the following events. Indicate which are simple and which are compound events.

a. One person is computer literate and the other is not.
b. At least one person is computer literate.
c. Not more than one person is computer literate.
d. The first person is computer literate and the second is not.

2. A hat contains 40 marbles. Of them, 18 are red and 22 are green. If one marble is randomly selected out this hat, what is the probability that this marble is:
a. Red
b. Green

3. A random sample of 2000 adults showed that 1120 of them have shopped at least once on the internet. What is the (approximate) probability that a randomly selected adult has shopped on the internet?

4. How many different outcomes are possible for 4 rolls of a fair 6-sided die?

5. A statistical experiment has 8 equally likely outcomes that are denoted by 1, 2, 3, 4, 5, 6, 7, and 8. Let event A={2, 5, 7} and event B={2, 4, 8}.

a. Are events A and B mutually exclusive events?
b. Are events A and B independent events?
c. What are the complements of events A and B, respectively, and their probabilities?

6. A small ice cream shop has 10 flavors of ice cream and 5 kinds of toppings for its sundaes. How many different selections of one flavor of ice cream and one kind of topping are possible?

7. Two thousand randomly selected adults were asked whether or not they had ever shopped on the internet. The following table gives a 2-way classification of the responses:

	Have Shopped	Have Never Shopped
Male	500	700
Female	300	500

a. If one adult is selected at random from these 2000 adults, find the probability that this adult

i. has never shopped on the internet

ii. is a male

iii. has shopped on the internet given that this adult is a female

iv. is a male given that this adult has never shopped on the internet

b. Are the events “male” and “female’ mutually exclusive? What about the events “have shopped” and “male”? Why or why not?

c. Are the events “female” and “have shopped” statistically independent? Why or why not?

8. A company hired 30 new college graduates last week. Of these, 16 are female and 11 are business majors. Of the 16 females, 9 are business majors. Are the events “female” and “business major” statistically independent? Are they mutually exclusive? Explain why or why not?

9. Find the joint probability (intersection) of A and B for the following:

a. $P(B)=0.59$ and $P(A|B)=0.77$

b. $P(A)=0.28$ and $P(B|A)=0.35$

10. Given that A, B, and C are 3 independent events, find their joint probability for he following:

a. $P(A)= 0.49$, $P(B)=0.67$ and $P(C)=0.75$

b. $P(A)=0.71$, $P(B)=0.27$, and $P(C)=0.45$

11. Given that $P(A)=0.65$ and $P(A \text{ and } B)=0.45$, find $P(B|A)$

12. The following table gives a 2-way classification of all basketball players at a state university who began their college careers between 1990 and 2000, based on gender and whether or not they graduated.

	Graduated	Did Not Graduate
Male	126	55
Female	133	32

a. If one of these players is selected at random, find the following probabilities
i. P(female AND graduated)
ii. P(male AND did not graduate)

b. Find P(graduated AND did not graduate)

13. The probability that an employee at a company is female is 0.36. The probability that an employee is a female AND married is 0.19. Find the conditional probability that a randomly selected employee from this company is married given that she is a female.

14. Given that A and B are 2 mutually exclusive events, find P(A OR B) for the following:

a. P(A)=0.25 and P(B)=0.27
b. P(A)=0.58 and P(B)=0.09

15. The probability that a family owns a washing machine is 0.68, that it owns a DVD player is 0.81, and that it owns both a washing machine and a DVD player is 0.58. What is the probability that a randomly selected family owns a washing machine or a DVD player?

Answers:
1. only d is a simple event
2. a. 0.450, b. 0.550
3. 0.560
4. 1296
5. a. no b. no c. not A={1, 3, 4, 6, 8}, not B={1, 3, 5, 6, 7}, P(not A)=0.625, P(not B)=0.625
6. 50
7. i. 0.600 ii. 0.600 iii. 0.375, iv. 0.583 b. yes and no. c. dependent
8. dependent but not mutually exclusive
9. a. 0.4543 b. 0.0980
10 a. 0.2462 b. 0.0863

11. 0.6923
12. i. 0.3844, ii. 0.1590 b. 0.0000
13. 0.5278
14 a. 0.52 b. 0.67
15. 0.910.

Chapter 5 Practice Problems

1. In a sample survey, 1800 senior citizens were asked whether or not they had ever been victimized by a dishonest telemarketer. The following table gives the responses by age group:

		Have Been Victimized (Y)	Have Never Been Victimized (N)
	60-69 years (A)	106	698
Age categories	70-79 years (B)	145	447
	80 or more yrs (C)	61	343

If 1 person is selected at random from these 1800 people, find the probability that this person:

a. Is 80 years or older

b. Has been victimized

c. Has been victimized GIVEN he/she is 70-79 years old

d. Is 60-69 years old GIVEN that he/she has not been victimized

e. Has never been victimized GIVEN that he/she is 80 year old or more

f. Is less than 80 years old

g. Is 70-79 years old AND has never been victimized

h. Is 70-79 years old OR has never been victimized

i. Are the events “60-69 years old” and “has been victimized” mutually exclusive? Why?

j. Are the events “80 years or older” and “60-69 years old” mutually exclusive? Why?

k. Are the events “70-79 years old” and “have been victimized” statistically independent? Why?

2. An appliance repair company that makes service calls to customers' homes has found that 5% of the time there is nothing wrong with the appliance and that the problem is due to customer error (appliance unplugged, controls improperly set, etc.). Two service calls are selected at random and it is observed whether or not the problem is due to customer error. Find the probability that in this sample of 2 service calls:

a. Both problems are due to customer error

b. At least one problem is NOT due to customer error.

3. A regular 6-sided die is rolled once. Event A occurs if an odd number is rolled. Event B occurs if a prime number is rolled. (Note: a prime number is one that can only be divided by itself and 1.) (Note that 1 is not considered a prime number)

a. What is the sample space S?

b. What is P(A)?

c. What is P(B)?

d. What is the complement of A?

e. What is the complement of B?

f. What is the complement of S?

g. List the elements of $A \cap B$ and determine $P(A \cap B)$.

h. List the elements of A U B and determine P(A U B).

i. What is $P(A|B)$?

j. Are A and B statistically independent? Why or why not?